LESSON 1: INTRODUCTION TO FRACTIONS

This lesson covers the following information:

- Understanding fractions
- Finding equal fractions
- Simplifying and expanding fractions
- Identifying improper fractions and converting it to a mixed number
- Identifying a mixed number and converting it to an improper fraction

Highlights include the following:

- Fractions are a part of a whole.
  - The denominator is the number of parts the whole unit is divided into.
  - The numerator is the number of those parts that are of interest.

- A proper fraction is a fraction in which the numerator is smaller than the denominator.
- Proper fractions represent quantities less than 1.
- An improper fraction is a fraction in which the numerator is larger than or equal to the denominator.
- Any number over itself equals 1.
- Any number over 1 equals the number.
- Fractions are called equivalent fractions if they represent the same quantity.
  - Multiply or divide the numerator and denominator of a fraction by the same nonzero number.
- A fraction is in simplest form, or reduced form, when the numerator and the denominator have no common factors other than 1.
  - To write a fraction in its simplest form (lowest terms), divide both the numerator and denominator by the greatest common factor (GCF) that divides evenly into both numbers.
- In some circumstances, we will need to write fractions so that they have a particular denominator. When we do this, we are said to expand fractions.
- To make an improper fraction a mixed number, divide the numerator by the denominator.
- To make a mixed number from an improper fraction, multiply the denominator times the whole number. Then, add the numerator to the product. Keep the denominator.

Reflection
Fractions represent parts of a whole and do not always have to be less than 1 (a whole). While proper fractions represent quantities smaller than 1, improper fractions represent quantities that are 1 or larger. If we multiply or divide both the numerator and denominator of a fraction by the same number (factor), the resulting fraction is equivalent to the original fraction. This property is used to both simplify (reduce) fractions and to expand fractions.
Notes:
Crossword Puzzle

Use the clues to solve the puzzle.

Across
6. The number in the bottom of a fraction.

Down
1. A fraction in which the numerator is smaller than the denominator. Its value is less than 1.
2. A fraction in which the numerator is larger than or equal to the denominator. Its value is 1 or larger.
3. The bar that divides the numerator from the denominator in every fraction. The fraction bar means “divided by”
4. The number in the top of a fraction.
5. A number used to represent part of a whole unit.
Practice Problems

1. Eight children went to the baseball game. Three children had popcorn and five children had candy. What fraction of the students had candy? ________________

2. Becca made 12 cupcakes for her coworkers. She put frosting on seven of them. What fraction of the cupcakes did not have frosting? ________________

3. Kendall had 20 plats of wood. His wife wanted to use five of the plats to complete a craft. What fraction of plats did Kendall have left? ________________

4. Elliot was doing a research for his science class. He was watching a heard of eight deer. Three deer ran away. What fraction of deer ran? ________________

5. Coryn received an arrangement of flowers. There were a dozen flowers and three were roses. What fraction of the flowers were roses? ________________

6. Bob bought bagels. Three were plain bagels, two were blueberry, and one was raisin. What fraction of the bagels were blueberry? ________________

7. The city has 100 restaurants. 50 of the restaurants serves pizza. Of these restaurants, only 30 are open on Monday nights. What fraction of restaurants that serves pizza is open on Monday? ________________

8. The local community center served lunch for school children during the summer months. There were 75 children who attend the lunch. 32 children drink milk and 43 children prefer juice. What fraction of children drink juice with lunch? ________________

9. An apple tree had 33 apples on the lowest branches. 12 apples fell to the ground. What fraction of apples remained on the tree? ________________

10. Alley joined a local theater group. There were 25 men and 23 women. What fraction of the group were women? ________________
LESSON 2: MULTIPLICATION WITH FRACTIONS

This lesson covers the following information:

- Multiplying fractions
- Using strategies to make multiplication easier

Highlights include the following:

- The rule for multiplying fractions is if $a$, $b$, $c$, and $d$ are numbers and $b$ and $d$ are not 0, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$
  - Multiply the numerators and multiply the denominators.
- When numerators and denominators have common factors when multiplying fractions, cancel the common factors from the numerator and denominator before multiplying.
- When multiplying fractions, if common factors are present between numbers in the numerator and numbers in the denominator cancel them prior to multiplying.
- When multiplying more than two fractions at a time, it is possible to cancel any numerator with any denominator. The fractions do not need to be next to each other to cancel.
- Since any numerator can be canceled with any denominator, any fraction that can be reduced to lowest terms can be reduced before canceling.

Reflection
When multiplying fractions, simply multiply the numerators and then multiply the denominators. However, when numerators and denominators have common factors, it is easier to cancel those common factors before multiplying.

Notes:
Word Search

Find all the words in the list. Words can be found in any direction.

CANCELLATION  COMMON
DENOMINATOR  FACTORS
FRACTION  NUMERATOR
RECIPIROCAL  REDUCED
SIMPLIFIED

N R O T A N I M O N E D
N O E D A R A I N O T E
L U I D N L M I O I F M
O A M T U R I T M T T F
U S C E A C I O M C T A
A C E O R L E O O A C C
E E T A R A L D C R T T
P D E O R P T E O F T O
R E D A O R I O C O R R
E E A A D R I C R N S S
S I M P L I F I E D A M
E T R A E A I O R R N C
Practice Problems

1. \( \frac{4 \cdot 4}{5 \cdot 5} = \) ____________________________

2. \( \frac{2 \cdot 6}{3 \cdot 8} = \) ____________________________

3. \( \frac{2 \cdot 7}{7 \cdot 10} = \) ____________________________

4. \( \frac{3 \cdot 6}{10 \cdot 8} = \) ____________________________

5. \( \frac{2 \cdot 4}{5 \cdot 7} = \) ____________________________

6. \( \frac{8 \cdot 4}{9 \cdot 10} = \) ____________________________

7. \( \frac{2 \cdot 6}{4 \cdot 9} = \) ____________________________

8. \( \frac{1 \cdot 2}{7 \cdot 8} = \) ____________________________

9. \( \frac{4 \cdot 2}{7 \cdot 7} = \) ____________________________

10. \( \frac{3 \cdot 3}{9 \cdot 4} = \) ____________________________
LESSON 3: DIVIDING FRACTIONS

This lesson covers the following information:

- Dividing fractions
- Identifying strategies for dividing fractions

Highlights include the following:

- Any two numbers are reciprocals of one another if their product is 1. So, 4 and $\frac{1}{4}$ are reciprocals because $\left(\frac{1}{4}\right)(4) = 1$

- Any number over 1 equals the number $\frac{5}{1} = 5$.

- When you are multiplying reciprocals, each numerator cancels with the denominator of the other fraction, leaving you with $\frac{7}{7} \cdot \frac{7}{7} = \frac{49}{49} = 1$.

- To find the reciprocal of a number, simply have to flip it over. So, if $a$ and $b$ are both real numbers other than 0, then the reciprocal of $\frac{a}{b} = \frac{b}{a}$.

- To divide by a fraction, multiply by the reciprocal.
  - To divide two fractions, flip the second fraction over and multiply the two fractions using canceling and reducing when possible.
  - This also works when dividing a whole number by a fraction (create a fraction $\frac{whole \ number}{1}$, flip the second fraction and multiply).

- When dividing a fraction by a whole number, create a fraction with the $\frac{whole \ number}{1}$, flip it, and multiply.

Reflection
You learned that you divide a fraction by multiplying by its reciprocal. You find the reciprocal of a fraction by simply flipping it over.

Notes:
Unit 2: Fractions and Mixed Numbers

Fallen Phrases

Unscramble the letters and solve the puzzle.

Practice Problems

1. \( \frac{1}{3} \div 2 = \) _____________________________

2. \( \frac{6}{11} + \frac{9}{11} = \) _____________________________

3. \( \frac{5}{8} \div \frac{11}{12} = \) _____________________________

4. \( \frac{2}{5} \div 3 = \) _____________________________

5. \( \frac{1}{2} \div \frac{1}{4} = \) _____________________________

6. \( 13 \div \frac{3}{4} = \) _____________________________

7. \( \frac{4}{5} \div \frac{13}{10} = \) _____________________________

8. \( \frac{5}{16} \div \frac{2}{3} = \) _____________________________
9. You have $\frac{2}{3}$ of a bag of candy and you want to divide it equally between 3 people. How much will each person get? ________________

10. Angie is making homemade salsa. Each batch calls for $\frac{1}{4}$ teaspoon of red pepper flakes. How many batches can she make if she has 6 teaspoons of red pepper flakes? ________________
LESSON 4: ADDING FRACTIONS

This lesson covers the following information:

- Determining the least common denominator (LCD)
- Adding fractions

Highlights include the following:

- A basic principle of addition is that we can only add two quantities if they are alike.
- The least common denominator (LCD) of a list of fractions is the smallest positive number divisible by all the denominators in the list.
- There are several strategies that will help in finding the LCD of any size numbers.
  - Determine if one of the denominators is a multiple of the other. If it is, then…
    - Convert the second fraction to the denominator of the multiple.
    - The LCD cannot be smaller than the largest denominator.
- Find the multiples of the largest denominator until you find a multiple that the other denominator(s) will divide evenly into.
- Convert the fractions to the LCD by multiplying both the numerator and denominator of each by the same factor.
- Another method for finding the LCD involves factoring each denominator.
  - Write the prime factorization of each denominator.
  - List all the factors that the two denominators have in common.
  - Add to the list the remaining factors from each denominator.
  - Multiply out the combined list of factors. The result is the least common denominator.
- If the two fractions to be added already have a common denominator, then simply add the numerators and write the result over the common denominator. Simplify/reduce the resulting fraction.
- If the fractions to be added do not have a common denominator, we must find the LCD, then convert each fraction to its equivalent fraction that has the common denominator. Add the fractions (with the new common denominator), and simplify the results.

Reflection
In this lesson, you learned that, in order to add two or more fractions, they must first have a common denominator. Therefore, if the fractions being added do not have a common denominator, then you must find the least common denominator (LCD), write the fractions as equivalent fractions with a denominator of the LCD, add the numerators and place the sum over the LCD, and simplify if possible.

Notes:
Letter Tiles

Unscramble the tiles to reveal a hidden message.

G I N D N O M N F T E S E R M O N T H
A D . S T C O M C A O F R O R T C I N
R E F M A N O T A M M O Y P E C O O F S T
L I N G E G R E A A C F C D O M M
R I T O R S T D E I N D S T E W H E T O R
L E A F A C B E I N A K E R R O A N E
U L O N T O . T H I

Practice Problems

1. Find the LCD of \( \frac{7}{12} \) and \( \frac{5}{6} \) = _______________
2. Find the LCD of $\frac{1}{6}$ and $\frac{5}{9}$. 

3. Find the LCD of $\frac{5}{7}$ and $\frac{2}{3}$. 

4. Find the LCD of $\frac{2}{7}$ and $\frac{1}{6}$. 

5. Find the LCD of $\frac{7}{4}$ and $\frac{9}{11}$. 

6. Find the LCD of $\frac{5}{12}$ and $\frac{3}{8}$. 

7. Sally wants to start walking every night. The first night she walks $\frac{3}{4}$ of a mile. The second night, she walks $\frac{3}{8}$ of a mile. How far did Sally walk? 

8. A group of community members want to create a garden and picnic area in a square grass lot. If they want to use $\frac{1}{4}$ of the land for the picnic area and $\frac{3}{8}$ of the area for the garden, how much space are they using? 

9. $\frac{2}{3}$ of Rosie’s patrons tipped at the end of their meal. $\frac{5}{6}$ of Shelly’s patrons tipped her. Which waitress received the most tips? 

10. Richard walked $\frac{1}{2}$ of a mile to work every morning. Then, he walked $\frac{2}{5}$ of a mile to the library. If he walked round trip each day, how far did Richard walk?
LESSON 5: SUBTRACTION WITH FRACTIONS

This lesson covers the following information:

• Subtracting fractions

Highlights include the following:

• If the two fractions to be subtracted already have a common denominator, then simply subtract the numerators and write the result over the common denominator. Simplify the resulting fraction, if possible.

• When the fractions to be subtracted do not have a common denominator, then just as when adding fractions, you must first find the LCD.

• Convert each fraction to its equivalent fraction that has the common denominator. Subtract the numerators of the like fractions, and simplify the result.

• A common error when subtracting fractions is to write the difference of the numerators over the difference of the denominators. This will not give correct results.

Reflection

As with addition of fractions, in order to subtract two fractions, they must first have a common denominator. Therefore, if the fractions being subtracted do not have a common denominator, then you must find the least common denominator (LCD), then write the fractions as equivalent fractions with a denominator of the LCD, subtract the numerators and place the difference over the LCD, and simplify if possible.

Notes:
Cryptogram

Solve the puzzle to view a hidden message.

Practice Problems

1. \[ \frac{11}{15} - \frac{7}{10} = \] ________________

2. \[ \frac{8}{9} - \frac{5}{12} = \] ________________

3. \[ \frac{15}{16} - \frac{3}{4} = \] ________________

4. \[ \frac{2}{3} - \frac{3}{10} = \] ________________
Unit 2: Fractions and Mixed Numbers

5. \( \frac{4}{5} - \frac{2}{3} = \) ________________________________

6. \( \frac{5}{6} - \frac{1}{4} = \) ________________________________

7. \( \frac{2}{3} - \frac{1}{6} = \) ________________________________

8. There was \( \frac{2}{3} \) inch of rain last month. This month, there was \( \frac{5}{6} \) inches of rain. How much more rain was there in the second month? __________________

9. Kris was studying his cell phone bill and he spent \( \frac{1}{15} \) of his minutes talking to his mother, and \( \frac{2}{3} \) of the minutes talking to his girlfriend. How much time did he spend talking to his best friend? __________________

10. Three children were sharing a box of cookies. The first student ate \( \frac{1}{2} \) the box. The second student ate \( \frac{2}{5} \) of the cookies. How much was left for the third child? __________________
LESSON 6: MIXED NUMBERS

This lesson covers the following information:

• Using mixed numbers to represent figures
• Converting mixed numbers to improper fractions
• Convert improper fractions to mixed numbers

Highlights include the following:

• A mixed number consists of a whole number and a fraction part.
• An improper fraction is a fraction in which the numerator is larger than or equal to the denominator and represents a quantity greater than or equal to 1.
• Mixed numbers can always be converted into improper fractions, and improper fractions can always be converted into mixed numbers.
• To convert a mixed number to an improper fraction, multiply the whole number by the denominator and add the numerator
• To convert an improper fraction into a mixed number, divide.
• The quotient will be the whole number of the mixed number and the remainder over the denominator of the original fraction will provide the fraction part of the mixed number.

Reflection
To count whole items or units, we use whole numbers. To describe part of a whole unit, use fractions. In many applications, however, a quantity may be expressed as a combination of the two. When we combine a whole number with a fraction, we obtain a mixed number.

Notes:
Word Search

Find all the words in the list. Words can be found in any direction.

DENOMINATOR  EQUAL TO  IMPROPER
FRACTION BAR   IMPROPER  MIXED NUMBER
LARGER THAN   WHOLE UNIT
SIMPLIFY

R O T A N I M O N E D
A E B I L M I W R R I
B E D E E P X H S I U
N Q E E M R E O I M I
O U T M E O D L M I I
I A E A A P N E P S I
T L E A M E U U L L O
C T R M A R M N I I A
A O T A A R B I F E X
R E E U R R E T Y O E
F L A R G E R T H A N
Practice Problems

1. Convert to a mixed number: \(\frac{61}{9}\) ________________
2. Convert to a mixed number: \(\frac{73}{8}\) ________________
3. Convert to a mixed number: \(\frac{76}{10}\) ________________
4. Convert to a mixed number: \(\frac{50}{5}\) ________________
5. Convert to a mixed number: \(\frac{47}{5}\) ________________
6. Convert to an improper fraction: \(9\frac{2}{13}\) ________________
7. Convert to an improper fraction: \(3\frac{9}{10}\) ________________
8. Convert to an improper fraction: \(12\frac{3}{5}\) ________________
9. Convert to an improper fraction: \(8\frac{4}{11}\) ________________
10. Convert to an improper fraction: \(6\frac{7}{15}\) ________________
LESSON 7: MULTIPLYING AND DIVIDING MIXED NUMBERS

This lesson covers the following information:

- Multiplying and dividing mixed numbers

Highlights include the following:

- Multiply mixed numbers in the same way that we multiply fractions except we first have to convert the mixed numbers to improper fractions.
- Express the final answer as a mixed number when possible.
  - Convert each mixed number to an improper fraction.
  - Cancel numerators with denominators.
  - Multiply all numerators.
  - Multiply all denominators.
  - Reduce/simplify if possible.
- Divide fractions by multiplying the fraction in front of the division symbol by the reciprocal of the fraction (flipped over) behind the division symbol.
  - To divide mixed numbers, first convert each mixed number into an improper fraction, then apply the property for dividing fractions.

Reflection
You have seen that multiplying and dividing mixed numbers is very similar to multiplying and dividing fractions. The only difference is that the mixed numbers must first be converted into improper fractions before the properties for multiplying and dividing fractions can be applied.

Notes:
Letter Tiles

Each block of letters is considered a tile. Unscramble the tiles to solve the phrase.

Practice Problems

1. Theresa has started her job at Hoyle farms. She was told to feed the cattle $4 \frac{1}{2}$ bales of hay every day. She needs to feed the horses $\frac{2}{3}$ times more hay than the cattle receive. How many bales of hay are the horses fed? ____________________
2. How many bales of hay with Theresa need for \( \frac{5}{2} \) days? ____________________

3. Last year, Suzanne was in charge of a salad lunch at her church. She ordered \( 3\frac{1}{3} \) batches of rolls. This year, she needed \( 1\frac{2}{3} \) times the original order. How many batches of rolls did she order? ____________________

4. There are two new trees in the yard. Ken wants to plant them about \( 12\frac{2}{3} \) feet apart. The oak tree is \( 3\frac{1}{3} \) feet tall. The walnut tree is \( 5\frac{1}{3} \) feet tall and the maple tree is \( 2\frac{5}{8} \) times as tall as the oak tree. How tall is the maple tree? ____________________

5. Jon and Carly started training for a race. Jon ran \( 2\frac{1}{2} \) miles in 6 minutes. Carly ran \( \frac{1}{3} \) time as many miles as Jon. How many miles did Carly run? ____________________

6. \( \frac{4}{2} + \frac{2}{5} = \) ____________________

7. \( \frac{4}{2} + \frac{3}{6} = \) ____________________

8. \( \frac{2}{3} + \frac{5}{2} = \) ____________________

9. \( \frac{3}{8} + \frac{3}{2} = \) ____________________

10. \( \frac{5}{10} + \frac{1}{2} = \) ____________________
LESSON 8: ADDING AND SUBTRACTING MIXED NUMBERS

This lesson covers the following information:
• Add mixed numbers
• Subtract mixed numbers

Highlights include the following:
• When adding mixed numbers, it is easier to write the problem vertically rather than horizontally. We convert the fraction parts to common denominators.
• First add the fraction parts and then the whole-number parts.
• In some cases, when we add the fraction parts of two mixed numbers, the result is an improper fraction.
• When this occurs, we convert this improper-fraction part of the sum into a mixed number and carry the whole-number part of the result to be combined with the whole number parts of the original mixed numbers.
• To subtract two mixed numbers, first subtract the fraction parts and then subtract whole parts together.
  ▪ As with addition of mixed numbers, we subtract mixed numbers by subtracting the similar parts. In other words, when subtracting mixed numbers, we first subtract the fraction parts and then the whole-number parts. We must subtract the fraction parts first in case it is necessary to borrow.

Reflection
As with multiplying and dividing, it is possible to add two or more mixed numbers by first converting each mixed number into an improper fraction. We can then proceed to follow the rules for adding fractions. However, the equivalent improper fractions can often involve very large numerators that are difficult to handle. Therefore, it is usually easier to add the similar parts. In other words, when adding and subtracting mixed numbers, it is usually easier to add the fraction parts together and the whole-number parts. You may have the following questions:

Notes:
Cryptogram

Solve the puzzle to reveal the hidden phrase.

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| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
| S | F | R | S | A | F | R | A | R | A | S | W | A | R | S | A | R | S | W | X | U | R | S | F | R | A |
| 8 | 10| 5  | 25| 8  | 17| 26| 11| 11| 17| 6 | 21| 10 | 25| 26| 19| 17| 5  | 12| 20|
| S | U | R | A | W | X | U | R | S | F | R | S | A | R | S | W | X | U | R | S | F | R | A |
| 10| 5  | 25| 8 | 17| 8 | 22| 24| 17| 25| 26| 19| 17| 17| 6 | 21| 10 | 25| 19| 17| 5  | 12| 20|
| A | S | U | R | A | W | X | U | R | S | F | R | S | A | R | S | W | X | U | R | S | F | R | A |
| 26| 20| 11| 17| 6 | 21| 20| 8 | 22| 24| 17| 25| 26| 19| 17| 17| 6 | 21| 1| 6 | 12| 14| 21| 20| 22| 4
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Practice Problems

1. \( \frac{9}{10} - \frac{3}{10} = \) ________________________________

2. \( \frac{3}{4} + \frac{1}{4} = \) ________________________________

3. \( \frac{2}{3} - \frac{1}{3} = \) ________________________________

4. \( \frac{1}{6} + \frac{5}{6} = \) ________________________________
Unit 2: Fractions and Mixed Numbers

5. \( \frac{2}{5} + \frac{4}{5} = \) ____________________________

6. \( \frac{7}{8} - \frac{3}{8} = \) ____________________________

7. \( \frac{1}{4} + \frac{2}{4} = \) ____________________________

8. \( \frac{9}{10} - \frac{7}{10} = \) ____________________________

9. \( \frac{2}{3} - \frac{1}{3} = \) ____________________________

10. \( \frac{7}{10} - \frac{1}{10} = \) ____________________________